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Layup Optimization Considering Free-Edge Strength and Bounded Uncertainty of Material Properties

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The layup optimization of the maximum strength of laminated composites with free edge by genetic algorithm (GA) is presented. To efficiently calculate the interlaminar stresses of composite laminates with free edges, the extended Kantorovich method was applied. In the formulation of GA, a repair strategy was adopted to satisfy given constraints. Multiple elitism schemes were implemented to efficiently find multiple global optima or near optima. A convex modeling technique is proposed to consider the bounded uncertainty of material properties. Results of the GA optimization with scattered properties were compared with those of optimization with nominal properties. The GA combined with convex modeling can work as a practical tool for lightweight design of laminated composite structures because geometric and material uncertainties are always encountered in composite materials.

Nomenclature

B	=	bending curvature with respect to the y axis
C	=	extensional strain along the x axis
E_i	=	uncertain material properties
E_i^0	=	nominal values of material properties
F, Ψ	=	Lekhnitskii stress functions
$f_i(\xi)$, $p_i(\xi)$	=	in-plane stress functions
$G(E_i)$	=	failure index
$g_i(\eta)$	=	out-of-plane stress functions
h	=	total thickness of the laminate
h_0	=	one-ply thickness
N	=	number of layers
N_e	=	number of elites to be copied into the next generation
n_j	=	surface normal vector
P_{cr}, P_{mu}	=	probability of crossover and or mutation
PopSize	=	size of population
R	=	reliability
S	=	shear strength
S_{ij}	=	compliances in the generalized coordinates
S_l	=	interlaminar tensile strength
u, v, w	=	displacement along the x or y or z axis
u_i	=	displacements
X_t, X_c	=	axial or longitudinal strength in tension or compression
Y_t, Y_c	=	transverse strength in tension or compression
α_i	=	coefficients of thermal expansion in the generalized coordinates
δ_i	=	deviation of material properties
ε_i, ε_{ij}	=	strain in the generalized coordinates
η, ξ	=	nondimensionalized coordinates in the z or y direction
Θ	=	relative angle of rotation about the x axis
σ_i, σ_{ij}	=	stress in the generalized coordinates
$\bar{\sigma}_{zz}$, $\bar{\sigma}_{zx}$	=	averaged interlaminar normal or shear stress

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I. Introduction

THE laminates with free edges suffer from the failure initiations by the critical interlaminar stresses near the free edges. Thus, if free edges cannot be avoided the layups should be arranged such that interlaminar stresses are minimized near the free edges of laminates. The interlaminar free-edge strength analysis plays a key role in solving the optimization problem of a layup design. Even though robust finite element method can provide accurate prediction of interlaminar stresses, it requires a large amount of computing time. In addition, the repeated efforts to prepare different meshes for analyzing the behaviors for various layup configurations make design optimization not only impossible but much harder. Thus, it is preferable to develop a simple and efficient analytical method to analyze the interlaminar stresses near the free edges. The extended Kantorovich method proposed by Cho and Yoon,[1] Cho and Kim,[2] and Kim et al.[3] demonstrated the efficiency and reliability of the method in predicting the free-edge interlaminar stresses and strength under thermomechanical loadings. The iterative extended Kantorovich method was used in the present study to analyze the strength of free edges.

In laminated composite structures the layups of laminates can be arranged for the lightweight and/or high performance of composite structures. Quite often in the structural layup design of composite rotor blades, the bending-torsion coupling is intentionally introduced to make the blade perform more efficiently. In most structural designs using composite laminates, laminates are restricted to some discrete sets of ply orientation angles such as 0, ±30, ±45, ±60, and 90 deg. This practical manufacturing point of view requires the discretized optimization methodology for the layup design problem.

Genetic algorithm (GA) or the simulated annealing method is considered an appropriate method for discretized optimization problems. Recently, a considerable number of researchers in optimization of design of the composite structures have reported using a GA. Comparatively, the application of the simulated annealing method is rare in this field. GA is a quite powerful methodology for the problem with integer variables and the problem in which the gradient of the objective functions is difficult to obtain. The application of GA was initially reported by Hajela[4] for composite structures. Harrison et al.[5] designed composite stringer stiffened panels by using a genetic algorithm. Le Riche and Haftka[6] proposed a GA to optimize the stacking sequence of composite laminate for maximum buckling load. For the same problem Liu et al.[7] provided permutation genetic algorithms. A recessive-gene-like repair strategy was introduced by Todoroki and Haftka[8] and by Todoroki and Sasai[9] to handle given constraints efficiently. Recently, Soremekun et al.[10] applied the generalized elitist selection (GES) to the problems with many global optima and those showing performance very close to optimal.

In the present study the layup optimization for the maximum strength of laminated composites with free edge is performed. The limitation in the selection of ply angles makes the layup design a combinatorial optimization problem. GAs have been used extensively to solve this type of combinatorial problems.[11–14] GAs are well suited for the problem of layup optimization, and because of random nature of GA they easily produce alternative optima in repeated runs. This property is particularly important in layup optimization because widely different layups can have very similar performance.[15]

Because of the uncertainty of the manufacturing process on the laminated composites, the stiffnesses, angle orientations, and ply thickness of laminates are not uniformly determined. The scattering of these parameters depends on the quality of cured laminates. These uncertainties should be included in the design process. However, the information on the overall probability density function for each of the parameters is difficult to obtain. Thus, we assume that the scattering bounds with respect to the nominal values of the parameters such as material properties are known priori. Then, by constructing a convex set from the scattering bound and sensitivity of the functional a modified functional considering the effect of uncertainty can be obtained.[16]

The objective of the present study is to consider this bounded uncertainty of material properties in the design optimization. The scattering bound, which depends on the manufacturing quality control, needs to be provided as input data. The variations of scattering bound will change the optimized layup configurations and maximum strengths.

The present paper consists of the following: First, the extended Kantorovich method for free-edge stress/strength analysis is outlined. Second, the formulation of a modified optimized functional subject to a convex set of constraints is derived. Third, a GA with a repair strategy and multiple elitism is outlined. Finally, numerical examples and discussions are provided.

II. Strength Analysis Considering Free-Edge Strength

A. Extended Kantorovich Method

The geometry of composite laminates with free edges under extension is given in Fig. 1. The laminate consists of orthotropic materials. The thickness of each ply is the same, and symmetric layups are considered. The linear elastic constitutive equations are assumed in each ply, and they are expressed in the following form:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} + \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ \alpha_6 \end{Bmatrix} \Delta T \tag{1}$$

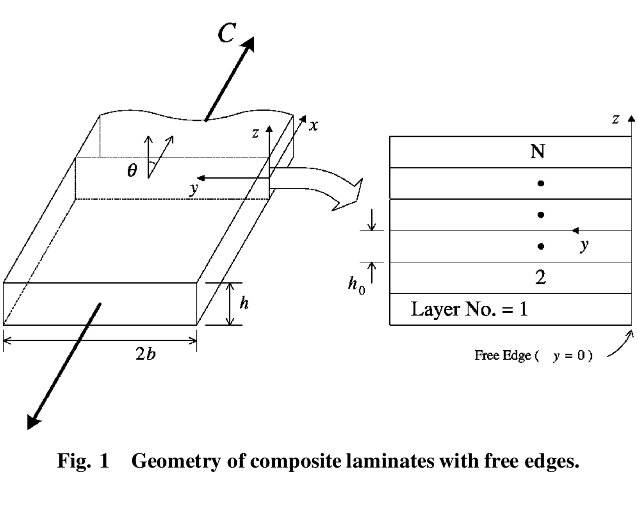

Fig. 1 Geometry of composite laminates with free edges.

For the given geometric configuration of laminates, the boundary conditions at the free edge and the surfaces of top and bottom faces are given in Eq. (2):

$$\sigma_2 = \sigma_4 = \sigma_6 = 0 \qquad \text{at} \qquad y = 0, 2b$$
$$\sigma_3 = \sigma_4 = \sigma_5 = 0 \qquad \text{at} \qquad z = \pm h/2 \tag{2}$$

Generalized plane strain states are assumed, and the stress fields are independent of x axis. The coordinates are nondimensionalized as follows:

$$\eta = z/h, \qquad \xi = y/h$$

Lekhnitskii stress functions are introduced to satisfy pointwise equilibrium equations automatically. These stress functions can be divided into the in-plane and out-of-plane functions. The functions $f_i(\xi)$ and $p_i(\xi)$ are in-plane functions, and $g_i(\eta)$ are out-of-plane functions. The individual stress components are obtained from Lekhnitskii stress functions, and the relationships are given as

$$\frac{\partial^2 F}{\partial \eta^2} = \sigma_2, \qquad \frac{\partial^2 F}{\partial \xi^2} = \sigma_3, \qquad \frac{\partial^2 F}{\partial \eta \partial \xi} = -\sigma_4$$
$$\frac{\partial \psi}{\partial \xi} = -\sigma_5, \qquad \frac{\partial \psi}{\partial \eta} = \sigma_6 \tag{3}$$

where

$$F = \sum_{i=1}^{n} f_i(\xi) g_i(\eta), \qquad \psi = \sum_{i=1}^{n} p_i(\xi) g_i^I(\eta) \tag{4}$$

The superscript I in Eq. (4) denotes differentiation with respect to η.

The in-plane stress functions are determined from the initially assumed basis set of out-of-plane functions, which must satisfy traction-free conditions at the top and bottom surfaces. The initial out-of-plane functions $g_i(\eta)$ are assumed to be the eigenmodes of a clamped–clamped beam vibration.

The governing equations are obtained from the principle of complementary virtual work:

$$\begin{aligned} 0 &= \iiint u_i \delta\sigma_{ij,j}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \\ &= \iiint \left\{ (u_i \delta\sigma_{ij})_{,j} - u_{i,j} \delta\sigma_{ij} \right\} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \\ &= \iint_s u_i \delta\sigma_{ij} n_j\, \mathrm{d}A - \iiint \frac{1}{2}(u_{i,j} + u_{j,i}) \delta\sigma_{ij}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z \end{aligned} \tag{5}$$

By using traction-free boundary conditions and neglecting rigid-body motions, one obtains

$$\iint (\Delta u \delta\sigma_{xx} + \Delta v \delta\sigma_{yx} + \Delta w \delta\sigma_{zx})\, \mathrm{d}y\, \mathrm{d}z = \iint \varepsilon_{ij} \delta\sigma_{ij}\, \mathrm{d}y\, \mathrm{d}z$$
$$(\Delta u = C - Bz, \quad \Delta v = -\Theta z, \quad \Delta w = B/2 + \Theta y) \tag{6}$$

Here, we only consider the extension behavior.

From the initially assumed basis function set, one can get the in-plane stress functions. The first process is given as follows. Substituting Eq. (3) into Eq. (6), the stresses are expressed in terms of f_i and p_i. The Euler differential equations for f_i and p_i can be obtained from Eq. (6). Thus, in-plane stress functions are determined from the initially assumed out-of-plane stress functions $g_i(\eta)$.

In the second process the Kantorovich method is reapplied to the original complementary virtual work given in Eq. (6). Substituting the in-plane stress functions f_i and p_i, which were obtained in the first process, into Eq. (6), the enhanced out-of-plane stress functions $g_i(\eta)$ are obtained by solving Euler equations given in Eq. (6).

The third iteration process is similar to the first one, and the fourth process is similar to the second process. In the computer program n-time iterations can be easily performed because the stress function patterns do not change after the second process. The detailed analysis process can be found in Refs. 1 and 2.

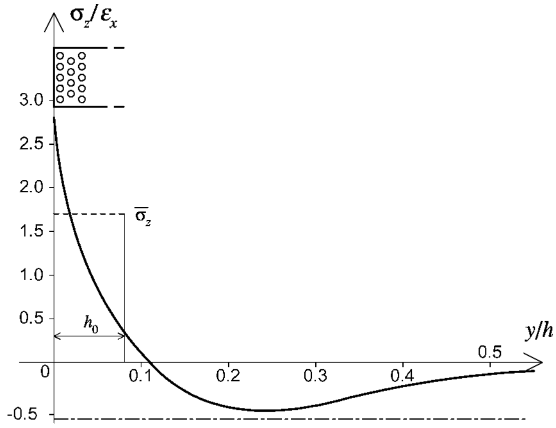

Fig. 2 Concept of averaging stress over the distance h_0 from the free edge.

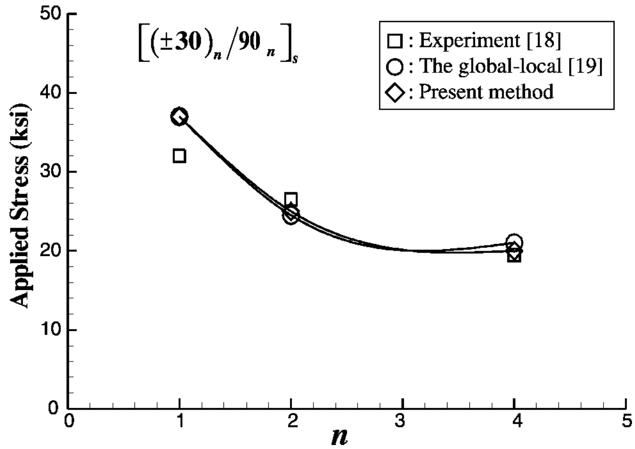

Fig. 3 Comparison of prediction and experiment for onset of delamination of the [±30n/90n]s laminate.

B. Strength Analysis by the Average Stress Criterion

The maximum averaged stress criterion proposed by Whitney and Nuismer[17] is employed for the evaluation of the proposed analysis method and compared with other known results of the interlaminar failure analysis. Failure occurs when the average value of interlaminar normal stress (σ_3) reaches interlaminar tensile strength S_l, that is, when

$$S_l = \frac{1}{h_0}\int_0^{h_0} \sigma(\xi, \eta)\,\mathrm{d}\xi \tag{7}$$

In Eq. (7) h_0 is taken as one ply thickness for all of the cases. The concept of the average stress $\bar{\sigma}_3$ is shown in Fig. 2.

The location and maximum value of interlaminar stress depend on the stacking sequences. That is, the failure is dominated by the interlaminar normal or shear stresses that are closest to the critical value. Kim and Soni[18] reported that failure is initiated by the interlaminar normal stress in the strength analysis of composite laminates they considered. The results of the present study were compared to the results of Kim and Soni.[18] T300/5208 graphite–epoxy composite laminates were considered, and the material properties are as follows:

$$E_1 = 137.9\ \text{GPa}, \qquad E_2 = E_3 = 9.65\ \text{GPa}$$

$$G_{12} = G_{13} = 5.52\ \text{GPa}, \qquad G_{23} = 4.14\ \text{GPa}$$

$$\nu_{12} = \nu_{13} = 0.3, \qquad \nu_{23} = 0.6$$

$$\alpha_1 = 22.28 \times 10^{-6}/^\circ\text{C}, \qquad \alpha_2 = -0.9 \times 10^{-6}/^\circ\text{C}$$

$$\Delta T = -111.1^\circ\text{C}$$

One of the results of strength analysis for composite laminates of which failure was dominated by the interlaminar normal stress is shown in Fig. 3. The maximum interlaminar normal stresses occurred at the midsurface of laminates. The experimental results agreed with the predicted values of the present method very well. The results of the present strength analysis were also correlated very well with the results of the global–local approach proposed in Ref. 19. The present results provide just a little conservative prediction of strength, but they were not distinguishable in the plottings shown in Fig. 3.

The present analysis method can provide reliable and efficient predictions of the maximal stress position through the thickness near the free edges, whereas the global–local approach in Ref. 19 might not indicate the maximal stress position properly through the thickness.

C. Maximum Stress Criterion and Quadratic Criterion

In the present layup optimization problem for maximal strength, a maximum stress criterion was adopted for the in-plane strength criterion, and a quadratic criterion[20] was used for the interlaminar strength criterion. They are given as

$$-X_c < \sigma_{xx} < X_t, \qquad -Y_c < \sigma_{yy} < Y_t, \qquad |\sigma_{xy}| < S \tag{8}$$

$$(\bar{\sigma}_{zz}/Y_t)^2 + (\bar{\sigma}_{zx}/S)^2 < 1 \tag{9}$$

where X and Y represent the failure strength along and transverse to the fiber direction, respectively. Tension and compression are represented by subscripts t and c, respectively. The reason for employing the quadratic criterion is that the criterion is more reliable and compact to apply than the independent maximum average stress criterion. According to the preceding criteria, we considered six failure modes: tensile axial, compressive axial, tensile transverse, compressive transverse, and interlaminar failure modes.

III. Considering Uncertainties in Material Properties: Convex Modeling

To consider the uncertainty of the design parameters, the function of the probability distributions should be known. However, the probability function of the scattering distribution requires sufficient data measurements. In the industry these uncertainty distribution functions might not be available. If the uncertainties under consideration are bounded with respect to the nominal reference values, a convex set from scattering bound of design parameters can be easily constructed. Convex modeling can be used in the constraint equations of the optimization problem to consider the uncertainties of material data and geometric data. This approach can be found in Refs. 16 and 21–23, and the procedure for analysis is outlined here to apply to our problems.

We consider only material uncertainties in the present study. Then $E_1 = E_L$, $E_2 = E_T$, $E_3 = \nu_{\mathrm{LT}}$, and $E_4 = G_{\mathrm{LT}}$. The failure index can be expanded up to linear terms by considering small parameter changes as follows:

$$G\left(E_i^0 + \delta_i\right) = G\left(E_i^0\right) + \sum_{i=1}^{4} \frac{\partial G\left(E_i^0\right)}{\partial E_i}\delta_i \tag{10}$$

Vectors $\{f\}$, $\{\delta\}$ are defined as follows:

$$\{f\}^T = \left[\frac{\partial G\left(E_i^0\right)}{\partial E_1}, \frac{\partial G\left(E_i^0\right)}{\partial E_2}, \frac{\partial G\left(E_i^0\right)}{\partial E_3}, \frac{\partial G\left(E_i^0\right)}{\partial E_4}\right] \tag{11}$$

$$\{\delta\}^T = [\delta_1, \delta_2, \delta_3, \delta_4] \tag{12}$$

Then the perturbed failure index can be symbolically given as

$$G\left(E_i^0 + \delta_i\right) = G\left(E_i^0\right) + \{f\}^T\{\delta\} \tag{13}$$

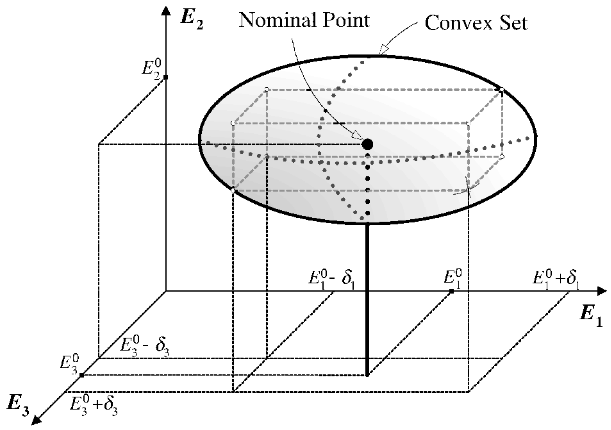

Fig. 4 Schematic of convex set in three-dimensional configurational space.

If it is assumed that δ_i construct a convex set, then from the linearity of Eq. (13) maximum values are on the boundary of the convex set. The constructed convex set of ellipsoid shape is derived as follows:

$$Z(e) = \left\{ \delta : \sum_{i=1}^{4} \frac{\delta_i^2}{e_i^2} \leq 1 \right\} \tag{14}$$

In Fig. 4 the schematic of the convex set in three-dimensional configurational space with three uncertain parameters E_1, E_2, and E_3 is displayed.

To obtain e_i, the following Lagrangian should be minimized:

$$L = Ce_1e_2e_3e_4 + \lambda\left(\Delta_1^2/e_1^2 + \Delta_2^2/e_2^2 + \Delta_3^2/e_3^2 + \Delta_4^2/e_4^2 - 1\right) \tag{15}$$

Through the variational process e_i are obtained as

$$e_i = 2\Delta_i \tag{16}$$

The problem of maximizing the failure index with material data scattering δ_i is constructed as the following form:

$$G_{\max} = \underset{\{\delta\} \in Z(e)}{\text{Max}} \left[G\left(E_i^0\right) + \{f\}^T\{\delta\}\right] \tag{17}$$

The problem can be expressed by the following modified Lagrangian:

$$L(\delta) = \{f\}^T\{\delta\} + \lambda(\{\delta\}^T\{\varepsilon\}\{\delta\} - 1) \tag{18}$$

where $\{\varepsilon\}$ is a diagonal matrix whose diagonal elements are $\varepsilon_{ii} = 1/e_i^2$.

After obtaining Lagrange multiplier $\{\delta\}$ for maximum failure index are obtained as

$$\{\delta\} = \pm 1\Big/\left(\sqrt{\{f\}^T\{\varepsilon\}^{-1}\{f\}}\right)\{\varepsilon\}^{-1}\{f\} \tag{19}$$

Maximum failure index considering bounded scattered material data can be finally obtained as

$$G_{\max} = G\left(E_i^0\right) \pm \sqrt{\{f\}^T\{\varepsilon\}^{-1}\{f\}}$$

$$= G\left(E_i^0\right) \pm \sqrt{\sum_{i=1}^{4}\left[e_i \frac{\partial G\left(E_i^0\right)}{\partial E_i}\right]^2} \tag{20}$$

IV. Genetic Algorithm

The design objective of the present study is to obtain layups of symmetric laminates that sustain the maximum applied load under just-mentioned independent maximum stress criterion. The 16-ply symmetric layup configurations are considered in the present study. Each ply thickness is fixed, and ply orientation angles are limited to 0, ±30, ±45, ±60, and 90 deg.

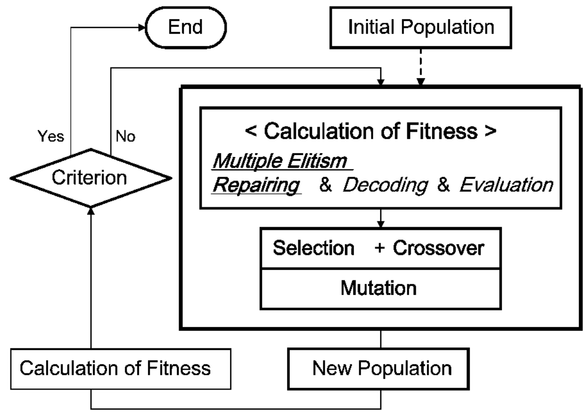

Fig. 5 Flowchart of GA.

Three constraints are applied to the present optimization problem. The first one is the symmetric layup constraint, but this is satisfied automatically by the coding rule that only half of the laminates are represented in a chromosome. The second constraint is a requirement of balanced laminate construction, which is intended to reduce or eliminate undesirable extensional–flexural coupling. The third constraint is a limit of four contiguous plies with the same fiber orientation, which reduces the problem of matrix cracking. The last two constraints are referred to as balance constraint and four-contiguity constraint, respectively, in the following descriptions. It is not easy to enforce these two constraints in genetic optimization. Penalty function can be used for handling these constraints. But in the present study a recessive-gene-like repair strategy, introduced by Todoroki and Haftka,[8] is applied with modifications. The key concept of the recessive-gene-like repair strategy is to repair the laminate without changing the chromosome.

For the problem with multiple global optima, the optimization process that finds as many optima as possible is required. To accomplish this requirement, multiple elitism strategies that copy the best designs in current generation into the next generation are adopted. This strategy is called by the first multiple elitist selection scheme among the GES procedures, which are proposed by Soremekun et al.[10]

A. Outline of GA Scheme

The flowchart of GA is illustrated in Fig. 5. To represent the ply angles in a layup as genes (in a chromosome), five numbers are introduced with each gene having one of the values of 0, 1, 2, 3, or 4. The gene-0 and gene-4 correspond to 0- and 90-deg plies, respectively. The first (outermost), third, fifth, etc. occurrences of gene-1 correspond to +30 deg, whereas even-number occurrences correspond to −30 deg. In similar way gene-2 and gene-3 represent ±45 and ±60 deg, respectively. Herein only half of the plies are represented by the chromosome because of the symmetry of laminates.

The initial population of chromosomes is generated at random. Each chromosome (laminate) is repaired by the following strategies and evaluated by the maximum applied load calculated by using the strength analysis described in the preceding section.

The N_e top chromosomes of each generation are always copied into the next generation by the multiple elitism. Selection is executed by a linear search through a roulette wheel with slots weighted in proportion to string fitness values.

After selection a two-point crossover, which is different from simple crossover, is conducted with a probability value of P_{cr}. Two random cutpoints are chosen first, and the offspring is generated by combining the middle segment of one parent with the outer segment of the other parent. When crossover is not conducted, the selected two parents are copied into the next generation. An example of a two-point crossover is given in Fig. 6.

Mutation is applied to the chromosomes, except for the elites of the previous generation. The probability of mutation P_{mu} is defined as the percentage of the total number of genes in the population. This

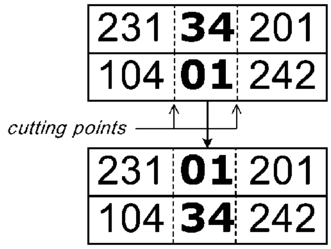

Fig. 6 Example of two-points crossover.

operator prevents premature loss of important genetic information by randomly altering a chromosome.

B. Repair Strategy for Balance Constraint

When the number of gene-1 is odd, the decoded laminate will be unbalanced, or only one unbalanced +30-deg ply will be in excess in the laminate. The situations will be same for gene-2 and gene-3. In the present study the strategy is classified into three cases. The procedures are adopted to make the least changes in the mechanical behavior of the repaired layup compared to the unrepaired layup.

When only one kind of gene among three genes (gene-1, gene-2, and gene-3) violates balance constraint, the innermost gene-1 is changed into gene-0, the gene-2 into gene-0 or gene-4 with the same probability, and the gene-3 into gene-4, respectively. If there is only one violating gene, the innermost gene-0 or gene-4 is altered to balance the violating gene. When two kinds of genes violate this constraint, the innermost gene among the violating genes is converted into the other kind of gene.

Consider a case in which there are three gene species that violate the balance constraint. If the innermost gene among the group of gene-1 and gene-3 is gene-1, the innermost violating gene-1 should be changed to gene-0. Next, the remaining violating genes are gene-2 and gene-3. The next repairing process is to convert the innermost violating gene (among the group of gene-2 and gene-3) into the remaining angle-ply gene (gene-2 ↔ gene-3). If gene-3 is the innermost instead of gene-1, the innermost gene-3 is converted into gene-4. Consecutively, the innermost violating gene (among the group of the gene-1 and gene-2) is switched into the remained angle-ply gene (gene-1 ↔ gene-2).

This strategy is similar to that of Todoroki's version but not the same. For example, in the present repairing strategy the repairing process for balance constraint precedes the four-contiguity constraint repairing process. Thus, there is no extra effort required to reconsider the four-contiguity constraint in checking the balance constraint. In addition, three kinds of angle-ply genes are considered in the present study (±30, ±45, ±60 deg), whereas only one kind of angle-ply gene is considered in Todoroki's study (±45 deg). Thus, the consideration of balance constraint is more complicated in the present strategy than in Todoroki's.

If the repair procedure does not change the laminate but changes the genes only, it can prevent beneficial changes that occur as a result of two or more consecutive mutations. For example, consider a case when it is beneficial to transform the chromosome [030030] corresponding to the $[0/60/0/0/\text{-}60/0]_s$ laminate into [040040] corresponding to the $[0/90/0/0/90/0]_s$ laminate. When the chromosome [030030] mutates into the chromosome [030040], the repair system reverses the change, and the chromosome still corresponds to the $[0/60/00/\text{-}60/0]_s$ laminate. In this case the repair procedure does not change the chromosome. One additional mutation in a future generation can transform the gene to [040040], and the innermost gene-2 will now be developed into the 90-deg ply. The innermost gene-2 acts like a recessive gene.

C. Repair Strategy for Four-Contiguity Constraint

This constraint is concerned with only the gene-0 and gene-4. When there are more than four contiguous genes, the innermost gene of the contiguous genes is converted into other gene. For example, $[02444420]_s$ is repaired into $[2444020]_s$.

For the innermost genes this repair procedure should be modified. When the two innermost genes are the same kind, there are already four contiguous plies with the same orientation in the middle of the laminate as a result of layup symmetry. Therefore, the repair process is not allowed to stack more than two genes in the innermost position within a laminate. When the innermost genes violate this rule, the gene value of the innermost ply is converted into the other gene. For example, $[04104100]_s$ is converted into $[04104104]_s$.

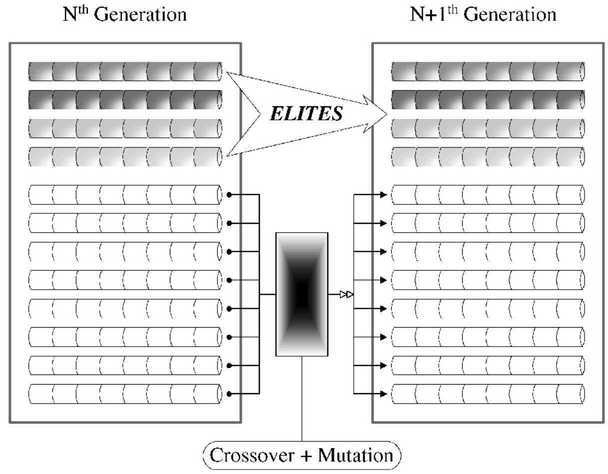

Fig. 7 Schematic of multiple elitism.

D. Multiple Elitism

The schematic of multiple elitism is shown in Fig. 7. The top designs (elites) from the parent population are selected and placed into the new population. The child designs required to fill the remainder of the new population are created from the remaining parents that have not been selected as multiple elites, and then placed into the new population. This selection scheme is computationally less intensive because fewer child designs require fitness computation. The number of elites to be copied into the next generation N_e are selected as follows according to population size:

$$N_e = \lfloor(\text{PopSize} + 5)/4\rfloor \tag{21}$$

where the symbol $\lfloor\ \rfloor$ (floor) indicates the largest integer smaller than or same as the number in the symbol. For example, when population size is 25 seven elites are selected and copied into the next generation.

V. Results and Discussion

In the present study graphite–epoxy composite laminates under the action of extensional loading are considered. We consider symmetric 16-ply layup configurations. The material properties and strength data are as follows:

$$E_1 = 207\ \text{GPa}, \qquad E_2 = E_3 = 5\ \text{GPa}$$

$$G_{12} = G_{13} = 2.6\ \text{GPa}, \qquad G_{23} = 1.8\ \text{GPa}$$

$$\nu_{12} = \nu_{13} = 0.25, \qquad \nu_{23} = 0.45$$

$$X_t = 1035\ \text{MPa}, \qquad X_c = 689\ \text{MPa}$$

$$Y_t = 41\ \text{MPa}, \qquad Y_c = 117\ \text{MPa}, \qquad S = 69\ \text{MPa}$$

In the calculation of the interlaminar stresses, three Kantorovich iteration processes are used, and two expansion terms, n in Eq. (4), are chosen. These selections are sufficient to calculate stresses reliably.

Various parameters (population size, probability of mutation, and probability of crossover) are given in Table 1. The numerical performance of GA is evaluated with respect to these parameters.

If the optimized results have been obtained, the performance of GA can be estimated for the various parameters. In the present study four different criteria are applied to assess the performance of the

Table 1 Parameters of GA

Parameter	Value
Chromosome length	8
Upper limit of generation	100
Number of runs N_r	30
Population size	7~50 (29)[a]
Probability of mutation	0.0~1.0 (0.1)[a]
Probability of crossover	0.0~1.0 (0.9)[a]

[a][( ) = default].

present GA in the layup optimization problem.[10] The first criterion is the normalized cost per genetic search C_n, determined by

$$C_n = (N_g N_c / R)(R = N_{\text{op}}/N_r) \tag{22}$$

where N_g is the number of generations per run and N_c is the number of child designs created in each generation. If GA is run N_r times and achieves success in finding at least one of several global optima N_{op} times of these runs, then the reliability R is calculated as the equation in the parentheses of Eq. (22).

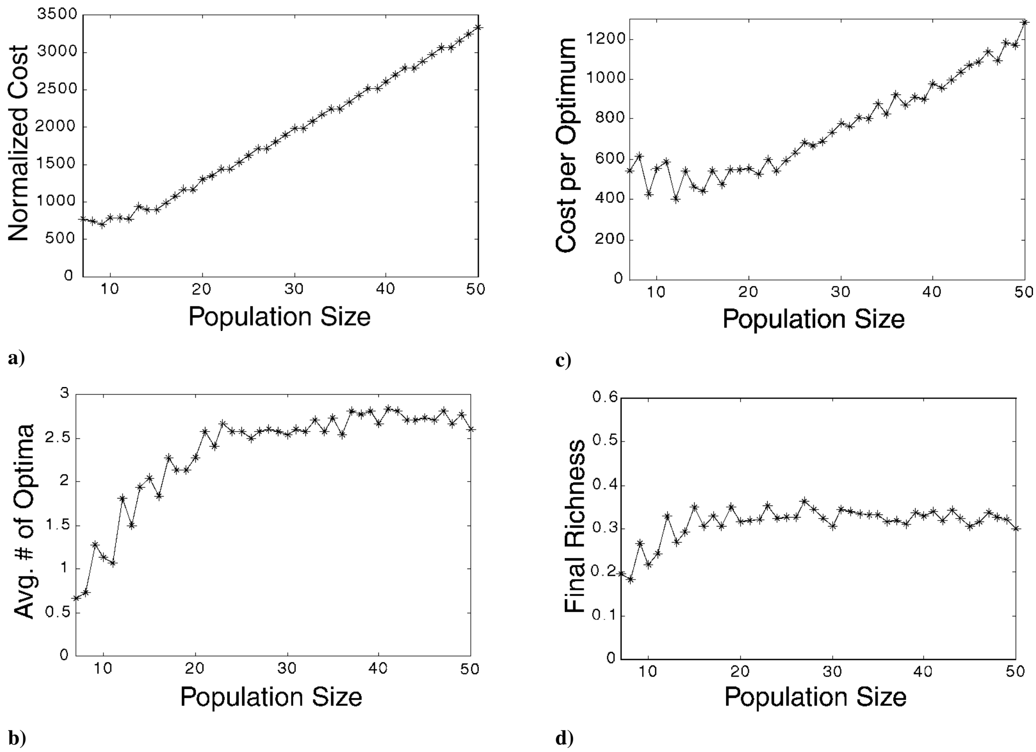

Fig. 8 Parametric study for population size (at nominal properties).

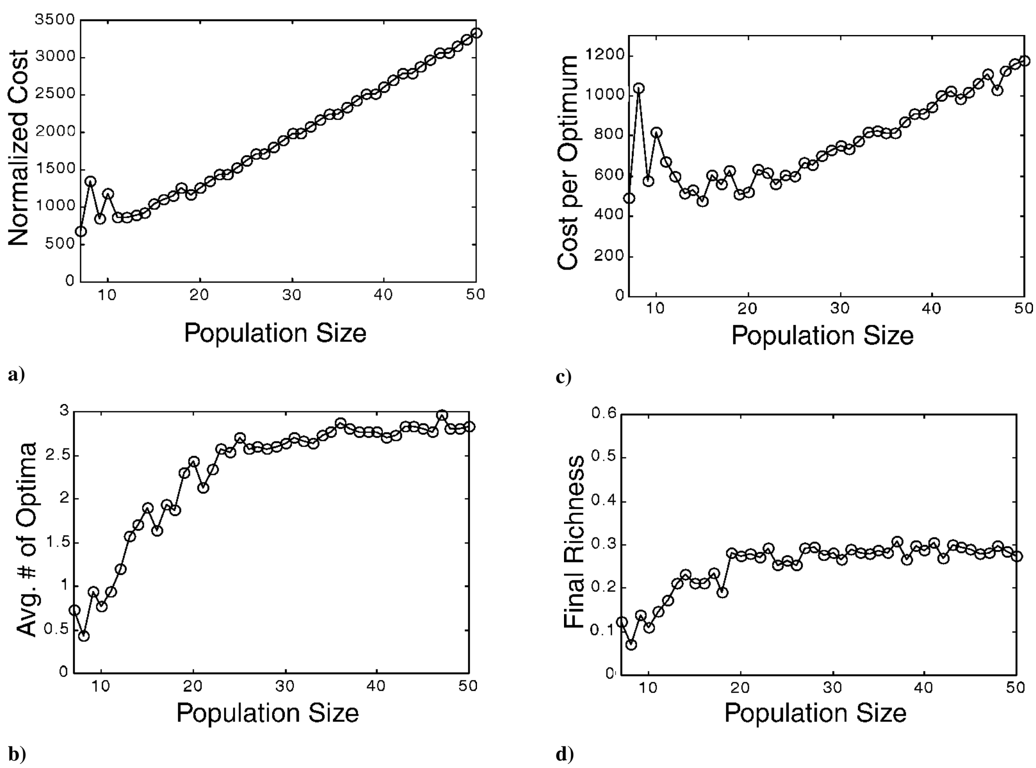

Fig. 9 Parametric study for population size (with uncertainty of properties).

The second criterion is the average number of optima found per genetic search:

$$A_{N_0} = \frac{\sum_{i=1}^{N_r} N_o^i}{N_r} \tag{23}$$

where N_o^i is the number of optima found in the ith optimization run. In the present problem we found out that the number of global optima is three through the extensive numerical simulations. Thus the maximum value of A_{N_o} is equal to 3.

The third criterion is defined as the cost per optimum found:

$$C_o = N_g N_c \big/ A_{N_o} \tag{24}$$

The final criterion is final population richness, which helps to monitor how the GA exploits global optimum regions of the design space. Final population richness P_r is defined as

$$P_r = N_{\Delta f} / (\text{PopSize} \cdot N_r) \tag{25}$$

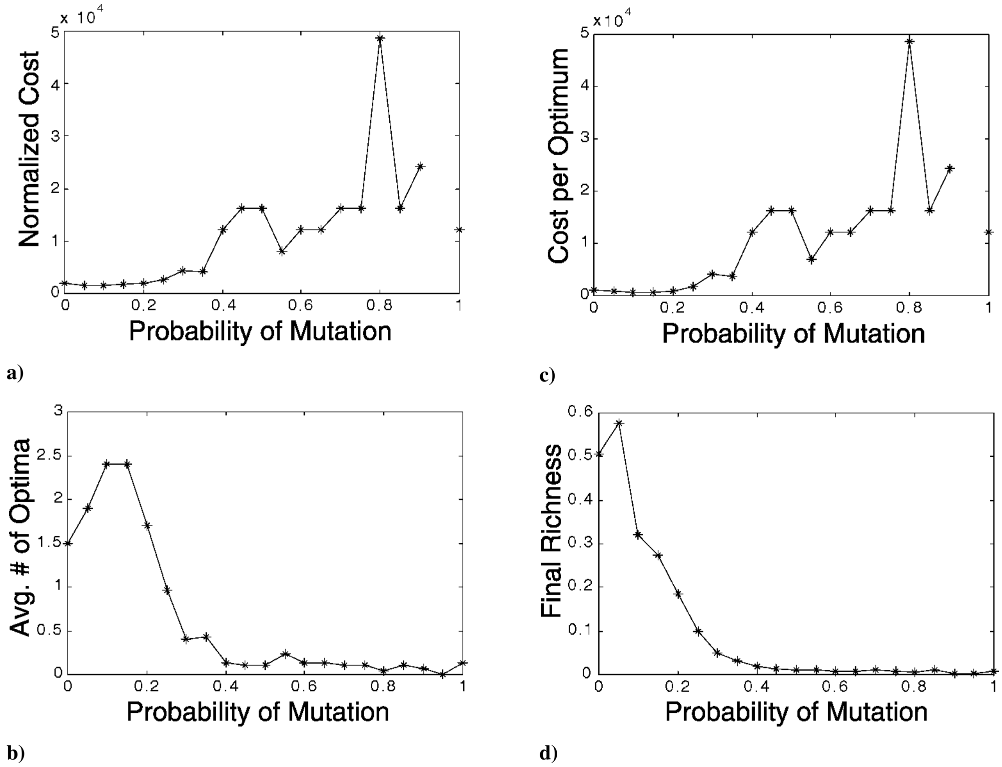

Fig. 10 Parametric study for probability of mutation (at nominal properties).

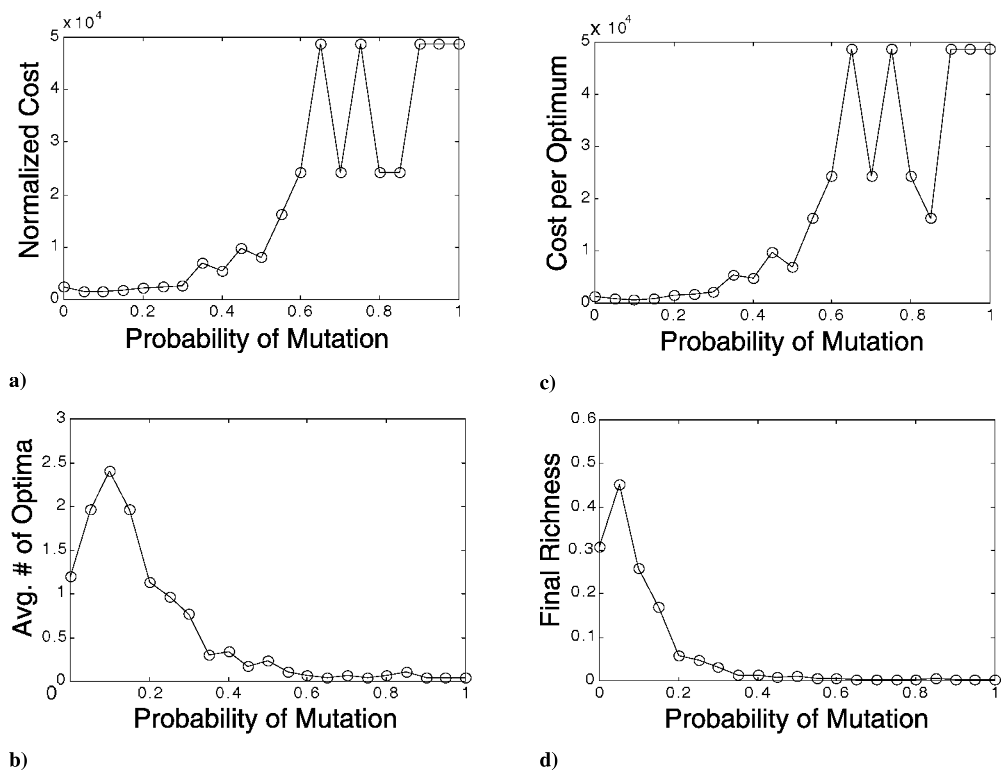

Fig. 11 Parametric study for probability of mutation (with uncertainty of properties).

where $N_{\Delta f}$ is the number of members in the final population of each run with fitness values within a certain small amount Δf of the optimum. In the evaluation of $N_{\Delta f}$, we consider the designs whose fitnesses are within the deviation of 7.7% from the optimal one.

In the present study the three global optima are obtained as $[0/0/0/30/0/0/0/-30]_s$, $[0/0/0/30/0/0/-30/0]_s$, and $[0/0/30/0/0/0/-30/0]_s$, regardless of the uncertainty in material properties. The optimal fitness is 6.915×10^5 for nominal properties. When the uncertainties of material properties are considered, the fitness is 6.877×10^5, which is 0.55% smaller than optimal fitness for nominal material values. The deviation of all of the material properties is set to 5% from nominal values. The failure mode of optimal design is tension in the axial direction for both cases.

The results of parametric evaluation in the application of GA are shown in Figs. 8–13. Figures 8, 10, and 12 are the plots using nominal material properties, and Figs. 9, 11, and 13 are the plots using scattered properties with 5% deviation from the nominal values. For a population size greater than 15, as the population size becomes larger, the GA is more costly but finds more optima. For this case final richness is nearly constant. Figures 10 and 11 show that the

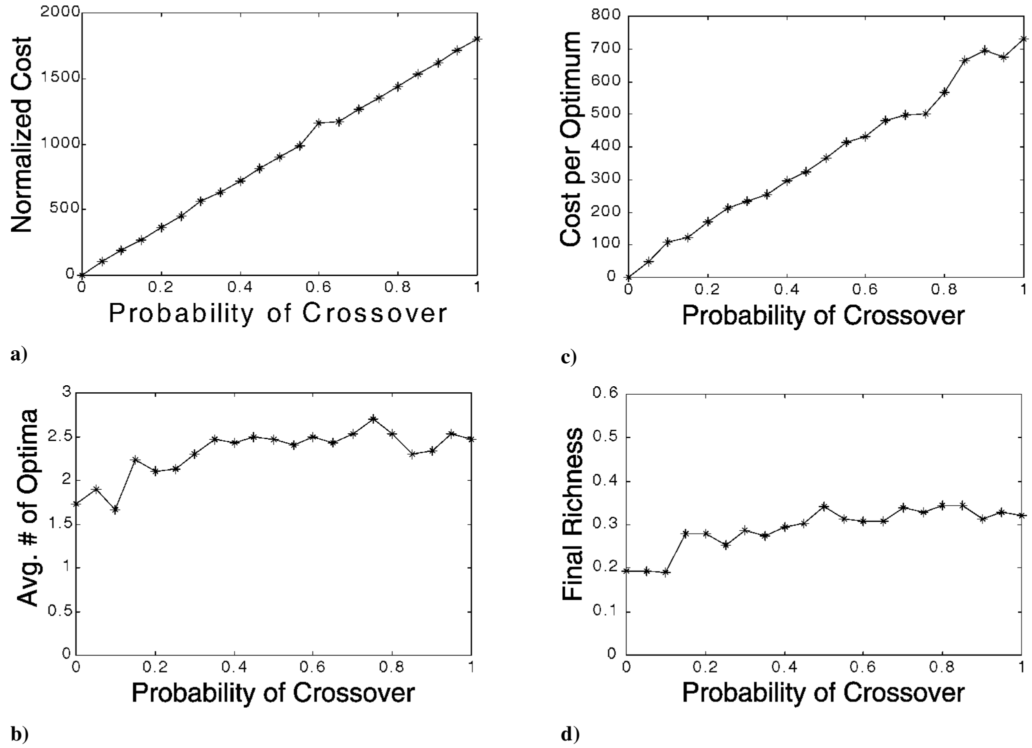

Fig. 12 Parametric study for probability of crossover (at nominal properties).

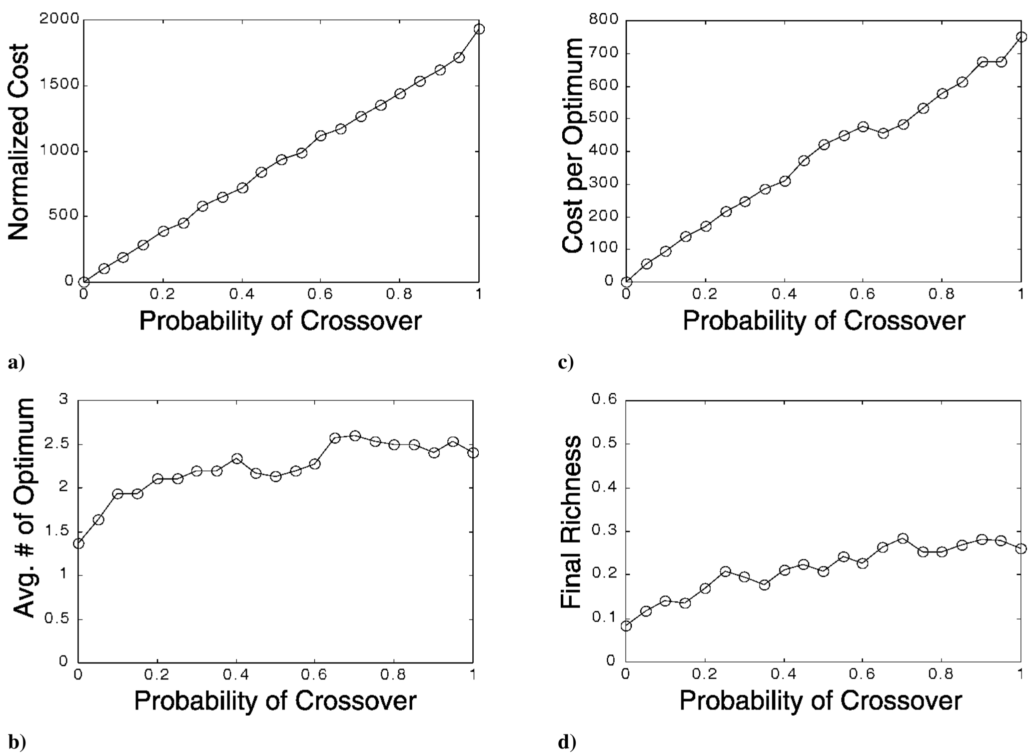

Fig. 13 Parametric study for probability of crossover (with uncertainty of properties).

Table 2 Sensitivities with respect to material properties

Material property	Normalized sensitivity
E_L	-2.936×10^4
E_T	$+1.982 \times 10^4$
ν_{LT}	$+0.961 \times 10^4$
G_{LT}	-0.457×10^4

cases with lower probability of mutation P_{mu} are better than those with higher probability, and the reliable range of the probability is from 0.05 to 0.15. As Figs. 12 and 13 show, the probability of crossover P_{cr} has less effects on the performance of GA than the probability of mutation does.

To construct a convex set for considering the bounded uncertainty of scattered material properties (E_L, E_T, ν_{LT}, and G_{LT}), the sensitivities are computed with a finite difference scheme. Or,

$$\frac{\partial G}{\partial E_i} = \frac{G^{\varepsilon} - G^0}{\varepsilon E_i} \qquad \begin{bmatrix} \varepsilon : \text{small number } (0.001) \\ E_i : \text{material properties} \end{bmatrix} \tag{26}$$

where G^0 and G^{ε} are the objective functions at the nominal properties and at the scattered properties, respectively.

When the convex modeling is applied even for the same layup, the failure mode changes as the amount of deviation of material properties is changed. Thus, six independent convex sets for one layup are constructed to consider six independent failure modes. The minimum among the failure indices calculated from the six convex sets is assigned as the fitness of the layup.

The sensitivities for optimal laminates are given in Table. 2. The strength sensitivity for each material property is given in the descending order. Modulus E_L has the largest sensitivity, and E_T is the second, and this result depends on the loading condition. In the extension problem changes of ν_{LT} and G_{LT} do not affect the strength of laminated composites significantly.

VI. Conclusions

The optimization by GA with repair strategy and multiple elitism for composite laminates was presented. It was demonstrated that GA with repair strategy is efficient in handling constraints. GA combined with multiple elitism is desirable for optimization; multiple designs with performance similar to that of global optimum can be found. To consider the bounded material property scattering, convex modeling was constructed. The bounded uncertainty can be easily considered in the optimization procedure by imposing the numeric values of the scattering bounds in the input process. The methodology proposed in the present study can be used as a powerful tool in the practical layup design of composite laminates. Interlaminar strength optimization can be extended to the various loading cases (bending, twisting, and thermal loads). This work is in the progress.

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